ROOT TEST

IF $\lim_{n\to\infty} \sqrt[n]{|a_n|} < 1$, THEN $\sum_{n=1}^{\infty} a_n$ CONV

IF $\lim_{n\to\infty} \sqrt[n]{|a_n|} > 1$ OR $\lim_{n\to\infty} \sqrt[n]{|a_n|} = \infty$, THEN $\sum_{n=1}^{\infty} a_n$ DIV

IF $\lim_{n\to\infty} \sqrt[n]{|a_n|} = 1$ OR DNE, NO CONCLUSION IS POSSIBLE WITHOUT OTHER ANALYSIS

eg. DOES $\sum_{n=1}^{\infty} \left(\frac{2n^2+5}{1-3n^2}\right)^{3n}$ CONV/DIV ?

$$\boxed{\lim_{n\to\infty} \sqrt[n]{\left|\left(\frac{2n^2+5}{1-3n^2}\right)^{3n}\right|}}$$

$$= \lim_{n\to\infty} \left(\left|\left(\frac{2n^2+5}{1-3n^2}\right)^{3\cancel{n}}\right|\right)^{\frac{1}{\cancel{n}}}$$

$$= \boxed{\lim_{n\to\infty} \left|\left(\frac{2n^2+5}{1-3n^2}\right)^{3}\right|}$$

$$= \boxed{\lim_{n\to\infty} \left|\left(\frac{2+\frac{5}{n^2}}{\frac{1}{n^2}-3}\right)^{3}\right|}$$

$$= \left|\left(\frac{2+0}{0-3}\right)^{3}\right|$$

$$= \left|\left(-\frac{2}{3}\right)^{3}\right|$$

$$= \boxed{\frac{8}{27}}\boxed{< 1} \rightarrow$$

$\sum_{n=1}^{\infty} \left(\frac{2n^2+5}{1-3n^2}\right)^{3n}$ $\boxed{\text{CONV (ROOT TEST)}}$

DOES $\sum_{n=1}^{\infty}\left(\frac{4n-1}{3-2n}\right)^n$ CONV/DIV ?

$$\boxed{\lim_{n\to\infty}\sqrt[n]{\left|\left(\frac{4n-1}{3-2n}\right)^n\right|}}$$

$$= \boxed{\lim_{n\to\infty}\left|\frac{4n-1}{3-2n}\right|}$$

$$= \boxed{\lim_{n\to\infty}\left|\frac{4-\frac{1}{n}}{\frac{3}{n}-2}\right|}$$

$$= \left|\frac{4-0}{0-2}\right|$$

$$= |-2|$$

$$= \boxed{2}\ \boxed{>1}$$

$\sum_{n=1}^{\infty}\left(\frac{4n-1}{3-2n}\right)^n$ $\boxed{\text{DIV (ROOT TEST)}}$

~~L'H~~ $\cancel{\frac{|4n-1|}{|3-2n|}}$

$\cancel{(|4n-1|)' \neq |4|}$

<u>11.7</u>

11.3 P-SERIES

11.2 GEOMETRIC
11.6 ROOT
11.2 DIVERGENCE

11.5 ALTERNATING/RATIO
11.5 ABS CONV
11.2 TELESCOPING

~~11.5 ABS CONV~~ ?

11.4 LIMIT COMP

11.4 COMP

11.3 INTEGRAL

11.5 ALT SERIES
ABS CONV
11.6 RATIO
ROOT

SHOULD YOU RUN RATIO TEST IF TERMS LOOK LIKE n^k ?

$\ln n$?

$$\lim_{n\to\infty} \frac{(n+1)^k}{n^k} = \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^k = (1+0)^k = 1$$

INCONCLUSIVE

SKIP RATIO TEST

$$\lim_{n\to\infty} \frac{\ln(n+1)}{\ln n} \stackrel{L'H}{=} \lim_{n\to\infty} \frac{\frac{1}{n+1}}{\frac{1}{n}}$$

$\frac{\infty}{\infty}$

$$= \lim_{n\to\infty} \frac{n}{n+1} = 1$$